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Advancements in ratio estimators: New linear combinations for estimating population mean

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Abstract
Sampling is the technique of selection of part of an aggregate population to represent the whole population and sample obtained is expected to be true representative of the whole population. In this paper, we have suggested ratio estimator using Tri-mean and the expression of bias and mean square error of the proposed estimator is derived up to first order approximation. An empirical study has been carried out to judge the merit of the suggested estimator over other existing estimators practically. From the results, the proposed estimator y_{pr6} having mean square error (2692.15) and % relative efficiency is 145.8%, which shows that the efficiency of the proposed estimator is more than the existing estimators.

Keywords: Ratio estimator, auxiliary variable, mean square error, percent relative efficiency

1. Introduction

Estimators are important in sampling theory because they allow researchers to estimate the characteristics of population based on a sample of that population. The main purpose of estimation in statistics is to be able to measure the behavior of data within a population. Ratio estimators take advantage of the correlation between the auxiliary variable, x and the study variable, y. The ratio estimator is a useful estimator to estimate the population mean when data on the auxiliary variable that is positively linked with the study variable are available. Recently, the use of supplementary information provided by auxiliary variables in survey sampling was extensively discussed [1-12]. For ratio estimators in sampling theory, population information of the auxiliary variable, such as the coefficient of variation, midrange, tri mean or the kurtosis, is often used to increase the efficiency of the estimation for a population mean. Among these estimators the ratio estimator and its modifications have widely attracted researchers throughout the world for the estimation of the mean of the study variable Kadilar and Cingi (2004, 2006a, b) [13], Subramani, (2013) [14], Subzar *et al*.(2018a, b) [15, 16], Subzar *et al*. (2019) [16], Khare and Khare (2019) [17], Priya and Tailor (2019) [18], Hafeez and Shabir (2015) [24], Hafeez *et al*. (2020) [25] and Hafeez *et al*. (2023) [26]. Raja., *et al*., (2023) [19] propose new modified ratio estimator using the linear combination of Co-efficient of Kurtosis and Tri- Mean of the auxiliary variable.

In this paper, we proposed some new ratio estimators using coefficient of variation, Kurtosis of auxiliary variate, Midrange, Tri mean and obtain mean square error (MSE) equations for all proposed estimators in Section III. We find theoretical conditions that make each proposed estimator more efficient than the traditional ratio estimators. In addition, we support these theoretical results with the aid of a numerical example.

2. The existing estimators

The classical ratio estimator for the population mean $\bar{Y}$ of the variate of interest y is defined by

$$\bar{y}_r = \bar{y}\frac{\bar{X}}{\bar{x}} \qquad (1)$$

where it is assumed that the population mean $\bar{X}$ of the auxiliary variate x is known. Here

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \; and \; \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ (2)

Where n is the number of units in the sample.
The MSE of the classical ratio estimator is given as

$$MSE(\bar{y}_r) = \frac{1-f}{n}\left(S_y^2 + R^2 S_x^2 - 2RS_{yx}\right) \quad (3)$$

Where
$f = \frac{n}{N}$; N is the number of units in the population;
$R = \frac{\bar{Y}}{\bar{X}}$ is the population ratio;
S_x^2 is the population variance of the auxiliary variate and
S_y^2 is the population variance of the variate of interest.

When the population coefficient of variation of auxiliary variate C_x is known, Sisodia and Dwivedi (1981) suggested a modified ratio estimator for $\bar{Y}$ as

$$\bar{y}_{SD} = \bar{y}\frac{\bar{X}+C_x}{\bar{x}+C_x} \quad (4)$$

MSE of the above estimator is given as:

$$\text{MSE}(\bar{y}_{SD}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\alpha\left(\alpha - 2\rho\frac{C_y}{C_x}\right)\right] \quad (5)$$

where C_y denote the coefficient of variation of Y, $\alpha = \frac{\bar{X}}{\bar{X}+C_x}$; ρ denote the correlation coefficient between Y and X.
Singh and Kakran (1993) proposed ratio estimator for $\bar{Y}$ as

$$\bar{y}_{SK} = \bar{y}\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)} \quad (6)$$

where
$\beta_2(x)$ is the population coefficient of Kurtosis of auxiliary variate.
MSE of this estimator was given as

$$MSE(\bar{y}_{SK}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\delta\left(\delta - 2\rho\frac{C_y}{C_x}\right)\right] \quad (7)$$

Where

$$\delta = \frac{\bar{X}}{\bar{X}+\beta_2(x)}$$

Upadhyaya and Singh (1999) suggested ratio estimators for $\bar{Y}$ as.

$$\bar{y}_{US1} = \bar{y}\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x} \quad (8)$$

$$\bar{y}_{US2} = \bar{y}\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)} \quad (9)$$

MSE of the two estimators were given as

$$MSE(\bar{y}_{US1}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\varepsilon\left(\varepsilon - 2\rho\frac{C_y}{C_x}\right)\right] \quad (10)$$

$$MSE(\bar{y}_{US2}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2\eta\left(\eta - 2\rho\frac{C_y}{C_x}\right)\right] \quad (11)$$

Where

$\varepsilon = \frac{\bar{X}\beta_2(x)}{\bar{x}\beta_2(x)+C_x}$, $\eta = \frac{\bar{X}C_x}{\bar{X}C_x+\beta_2(x)}$

We authors propose two new estimators using the linear combinations of coefficient of variation and Tri-mean.

$$\bar{y}_{OS1} = \bar{y}\frac{\bar{X}+C_xTM}{\bar{x}+C_xTM} \quad (12)$$

MSE of the above estimator is given as

$$MSE(\bar{y}_{OS1}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2 z\left(z - 2\rho\frac{C_y}{C_x}\right)\right]$$

where C^y denote the coefficient of variation of Y, $Z = \frac{\bar{X}}{\bar{x}+C_xTM}$.

TM is tri mean and ρ denote correlation coefficient between Y and X.

$$\bar{y}_{OS2} = \bar{y}\frac{\bar{X}+C_xMR}{\bar{x}+C_xMR} \quad (13)$$

$$MSE(\bar{y}_{OS2}) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_x^2 k\left(k - 2\rho\frac{C_y}{C_x}\right)\right]$$

Where C_y denote the coefficient of variation of Y, $k = \frac{\bar{X}}{\bar{x}+C_xMR}$

MR is mid-range and ρ denote correlation coefficient between Y and X.

3. The proposed estimators

When $\bar{y}$ in (4) is replaced with $\bar{y}_r$, the proposed ratio estimator based on coefficient of variation and tri mean is obtained as

$$\bar{y}_{pr1} = \bar{y}_r\frac{\bar{X}+C_x-TM}{\bar{x}+C_X-TM} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}+C_x-TM}{\bar{x}+C_X-TM} \quad (14)$$

MSE of this estimator can be found using Taylor series method defined as

$$f(\bar{y},\bar{x}) \cong f(\bar{Y},\bar{X}) + \left.\frac{\partial f}{\partial \bar{y}}\right|_{(\bar{Y},\bar{X})}(\bar{y}-\bar{Y})\left.\frac{\partial f}{\partial \bar{x}}\right|_{(\bar{Y},\bar{X})}(\bar{x}-\bar{X}) \quad (15)$$

Where

$$f(\bar{y},\bar{x}) = \bar{y}_{pr1}$$

$$\bar{y}_{pr1} - \bar{Y} \cong (\bar{y} - \bar{Y}) - \left(\frac{\bar{Y}}{\bar{X}} + \frac{\bar{Y}}{\bar{X}+C_x}\right)(\bar{x} - \bar{X})$$

$$= (\bar{y} - \bar{Y}) - (R + \lambda_1)(\bar{x} - \bar{X})$$

Where

$$\lambda_1 = \frac{\bar{Y}}{\bar{X}+C_x}$$

MSE of this estimator was given as follows:

$$MSE(\bar{y}_{pr1}) = E(\bar{y}_{pr1} - \bar{Y})^2$$

$$\cong E\begin{bmatrix} (\bar{y}-\bar{Y})^2 + (R+\lambda_1)^2 \\ +(\bar{x}-\bar{X})^2 - 2(R+\lambda_1) \\ (\bar{y}-\bar{Y})(\bar{x}-\bar{X}) \end{bmatrix}$$

$$= \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_1)^2 S_x^2 \\ -2(R+\lambda_1)S_{yx} \end{bmatrix} \quad (16)$$

When $\bar{y}$ in (6) is replaced with $\bar{y}_r$, the proposed ratio estimator is obtained as

$$\bar{y}_{pr2} = \bar{y}_r \frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)} \quad (17)$$

When Taylor series method is used for this estimator in the same way to obtain its MSE equation, we define

$f(\bar{y},\bar{x}) = \bar{y}_{pr2}$ in (15). Therefore,

$$MSE(\bar{y}_{pr2}) = \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_2)^2 S_x^2 \\ -2(R+\lambda_2)S_{yx} \end{bmatrix} \quad (18)$$

Where
$\lambda_2 = \frac{\bar{Y}}{\bar{X}+\beta_2(x)}$.

When $\bar{y}$ in (8) and (9) are replaced with $\bar{y}_r$, respectively, we also propose the ratio estimators as follows:

$$\bar{y}_{pr3} = \bar{y}_r \frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}\beta_2(x)+C_x}{\bar{x}\beta_2(x)+C_x} \quad (19)$$

$$\bar{y}_{pr4} = \bar{y}_r \frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}C_x+\beta_2(x)}{\bar{x}C_x+\beta_2(x)} \quad (20)$$

Similar to (16) and (18), MSE of these two estimators were given as follows:

$$MSE(\bar{y}_{pr3}) = \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_3)^2 S_x^2 \\ -2(R+\lambda_3)S_{yx} \end{bmatrix} \quad (21)$$

$$MSE(\bar{y}_{pr4}) = \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_4)^2 S_x^2 \\ -2(R+\lambda_4)S_{yx} \end{bmatrix} \quad (22)$$

where $\lambda_3 = \frac{\bar{Y}}{\bar{X}\beta_2(x)+C_x}$, $\lambda_4 = \frac{\bar{Y}}{\bar{X}C_x+\beta_2(x)}$

$$\bar{y}_{pr5} = \bar{y}_r \frac{\bar{X}+C_xTM}{\bar{x}+C_x\,TM} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}+C_xTM}{\bar{x}+C_xTM} \quad (23)$$

$$\bar{y}_{pr6} = \bar{y}_r \frac{\bar{X}+C_xMR}{\bar{x}+C_x\,MR} = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{X}+C_xMR}{\bar{x}+C_xMR} \quad (24)$$

MSE of the above estimators is given by

$$MSE(\bar{y}_{pr5}) = \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_5)^2 S_x^2 \\ -2(R+\lambda_5)S_{yx} \end{bmatrix} \quad (25)$$

$$MSE(\bar{y}_{pr6}) = \frac{1-f}{n}\begin{bmatrix} S_y^2 + (R+\lambda_6)^2 S_x^2 \\ -2(R+\lambda_6)S_{yx} \end{bmatrix} \quad (26)$$

Where

$$\lambda_5 = \frac{\bar{Y}}{\bar{X}+C_x TM}, \lambda_6 = \frac{\bar{Y}}{\bar{X}+C_x MR}$$

4. Efficiency comparisons

If we compare the MSE of the proposed estimators given in Eqns. (14), (16), (19), (20), (23), (24) with the MSE of the traditional ratio estimator given in Eq. (3) we have the conditions

$$MSE(\bar{y}_{pri}) < MSE(\bar{y}_r)\ (i = 1,2,3,4,5,6)$$

$$\Leftrightarrow \frac{1-f}{n}[S_y^2 + (R+\lambda_i)^2 S_x^2 - 2(R+\lambda_i)S_{yx}]$$

$$< \frac{1-f}{n}[S_y^2 + R^2 S_x^2 - 2RS_{yx}]$$

$$\Leftrightarrow \lambda_I(\lambda_I S_x + 2RS_x - 2\rho S_y) < 0. \quad (27)$$

where ρ is the correlation coefficient between auxiliary and study variables.

5. Numerical illustration

We use the data of Sarandal (1992) given in page no. 653 and the data statistics is given as

Table 1: Data statistics

$N = 117$	$\rho = 0.9917$	$\beta_1 = 0.20$
$n = 15$	$S_y = 862$	$\beta_2 = 1.10$
$\bar{Y} = 2179$	$S_x = 235.5$	TM $= 562.0$
$\bar{X} = 560.0$	$C_x = 0.7395$	MR $= 550.0$

MSE of the traditional ratio estimator and proposed estimators can be seen in Table II.

Table 2: MSE values of ratio estimators and percentage relative efficiencies

Estimators	MSE	PRE
y_r	3925.16	100
y_{pr1}	3598.5	109.07
y_{pr2}	3180.7	123.40
y_{pr3}	2861.5	137.17
y_{pr4}	2910.30	134.87
y_{pr5}	2720.14	144.29
$\boldsymbol{y_{pr6}}$	2692.15	145.80

From table II, we understand that the most efficient estimator is y_{pr6}. Therefore, the estimator with the linear combination of mid-range performs better among all the existing estimators.

6. **Conclusion**

We develop some ratio estimators of a finite population mean using auxiliary variable and theoretically show that the proposed estimators are more efficient than the traditional estimator in certain conditions. These theoretical conditions are also satisfied by the results of a numerical example.

7. References

1. Kadilar C, Cingi H. Improvement in estimating the population mean in simple random sampling. Appl Math Lett. 2006;19:75-79.

2. Kadilar C, Candan, Cingi H. Ratio estimators using robust regression. Hacettepe J Math Stat. 2007;3(2):181-188.
3. Kadilar C, Cingi H. A new estimator using two auxiliary variables. Appl Math Comput. 2005;162:901-908.
4. Kadilar C, Cingi H. An improvement in estimating the population mean by using the correlation coefficient. Hacettepe J Math Stat. 2006;35(1):103-109.
5. Kadilar C, Cingi H. A study on the chain ratio-type estimator. Hacettepe J Math Stat. 2003;32:105-108.
6. Gupta S, Shabbir J. On the use of transformed auxiliary variables in estimating population mean by using two auxiliary variables. J Stat Plan Inference. 2007;137:1606-1611.
7. Kadilar C, Cingi H. Ratio estimators in stratified random sampling. Biometrical J. 2003;45(2):218-225.
8. Kadilar C, Unyazici Y, Cingi H. Ratio estimator for the population mean using ranked set sampling. Stat Pap. 2009;50:301-309.
9. Rao TJ. On a class of almost unbiased ratio estimators. Ann Inst Stat Math. 1981;33:225-231.
10. Upadhyaya LN, Singh HP. Using of transformed auxiliary variable in estimating the finite population mean. Biometrical J. 1999;41(5):627-636.
11. Wei LD, Yan ZZ, Hong ZM. Ratio estimation with multivariate auxiliary variable linear combination. Appl Stat Manag. 2009;28(3):443-448.
12. Olkin I. Multivariate ratio estimation for finite populations. Biometrika. 1958;45:154-165.
13. Kadilar C, Cingi H. Ratio estimators in simple random sampling. Appl Math Comput. 2004;151:893-902.
14. Subramani J. Generalized modified ratio estimator for estimation of finite population mean. J Mod Appl Stat Methods. 2013;12(2):121-155.
15. Subzar M, Maqbool S, Raja TA, Abid M. Ratio estimators for estimating population mean in simple random sampling using auxiliary information. Appl Math Inf Sci Lett. 2018;6(3):123-130.
16. Subzar M, Maqbool S, Raja TA, Pal SK, Sharma P. Efficient estimators of population mean using auxiliary information under simple random sampling. Stat Transit New Ser. 2018;19(2):219-238.
17. Khare BB, Khare S. On the utilization of known coefficient of variation and preliminary test of significance in the estimation of population mean. Int J Agric Stat Sci. 2019;15(1):35-37.
18. Mehta P, Tailor. R estimation of ratio of two population means in stratified random sampling. Int J Agric Stat Sci. 2019;15:199-203.
19. Raja TA, *et al*. On ratio estimation using auxiliary information in survey sampling. Int J Agric Sci. 2023;15(2):12199-12203.
20. Sisodia BV, Dwivedi VK. A modified ratio estimator using coefficient of variation of auxiliary variable. J Indian Soc Agric Stat. 1981;33:13-18.
21. Singh HP, Kakran MS. A modified ratio estimator using known coefficient of kurtosis of an auxiliary character; c1993. (unpublished).
22. Upadhyaya LN, Singh HP. Use of transformed auxiliary variable in estimating the finite population mean. Biometrical J. 1999;41(5):627-636.
23. Särndal C-E, Swensson B, Wretman JH. Model assisted survey sampling. New York: Springer; c1992.
24. Hafeez W, Shabbir J. Estimation of the finite population mean, using median-based estimators in stratified random sampling. J Stat Appl Prob. 2015;4(3):367.
25. Hafeez W, Shabbir J, Shah MT, Ahmed S. Some median-type estimators to estimate the finite population mean. Asian J Prob Stat. 2020;7(4):48-58.
26. Hafeez W, Shabbir J, Aziz N. Estimation of finite population mean by using median-type estimators in two-stage sampling. Thailand Stat. 2023;21(1):370-382.